Also, A_3, B_3, C_3 and A_4, B_4, C_4 are on the perpendiculars at the middle points of the respective sides of the triangle ABC. Since the Simson lines to Q_3 and Q_4 correspond to the extremities of a diameter, they are perpendicular to each other, and therefore their parallels $A_3B_3C_3$ and $A_4B_4C_4$ are also perpendicular to each other.

Furthermore, $Q_{3a}M_a=Q_{4a}M_a$,

$$Q_{3a}M_a : M_aK_a=Q_{4a}M_a : M_aK_a,$$

$$Q_{3a}M_a : M_aK_a=Q_{3a}A_3 : A_3K=A_3M_a : A_3A_1,$$

and
$$Q_{4a}M_a : M_aK_a=Q_{4a}A_4 : A_4K=A_4M_a : A_4A_1,$$

or
$$A_3M_a : A_3A_1=A_4M_a : A_4A_1,$$

whence $\{M_aA_1,\ A_3A_4\}$ is an harmonic range, and $E\{M_aA_1,\ A_3A_4\}$ is an harmonic pencil. Since $\measuredangle A_4EA_3=90^\circ$, EA_3 will bisect the angle A_1EM_a.

Now, in two similar triangles the bisectors of the angles formed by any line in one triangle with the corresponding line in the other triangle are parallel to each other, hence the bisector of the angle formed by A_1E and EM_a, or the line AE, *i. e.*, the line $A_3B_3C_3$,—which is parallel to the Simson line belonging to one of the points of intersection of Brocard's Diameter, and the circumcircle about the triangle ABC,—is parallel to the bisector of the angle formed by B_1, C_1, and BC. (For particulars I can refer to my Geometrical Treatment of curves which are isogonal conjugate to a straight line with respect to a triangle, published by Leach, Shewell and Sanborn, New York.)

An excellent solution of this problem was also received from Professor G. B. M. Zerr.

CALCULUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

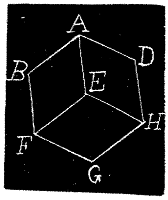
36. Proposed by H. C. WHITAKER, B. Sc., M. E., Professor of Mathematics, Manual Training School, Philadelphia, Pennsalvania.

A cube is revolved on its diagonal as an axis. Define the figure described and calculate its volume.

III. Solution by the PROPOSER.

The $\triangle$ BDE has each side$=\sqrt{2}a$, hence the radius of its circumscribed circle$=\frac{1}{3}\sqrt{6}a$. Hence the distance of A to the plane of $BDE=\frac{1}{3}\sqrt{3}a$. Take the origin at the center of the cube and the line AG as the axis of Z. The revolution will bring each line of the gauche hexagon $EHDCBF$ into either

the position of DH or BF. The equations of DH are $x=\frac{1}{2}\sqrt{2}$, $y=-\sqrt{2}z$, and the equations of BF are $x=-\frac{1}{2}\sqrt{2}$, $y=-\sqrt{2}z$. In either case $x^2=\frac{1}{2}$ and $y^2=2z^2$ and $x^2+y^2=2z^2+\frac{a^2}{2}$ which is the equation of the surface generated by the gauche hexagon $EHDCBF$. This surface could also be generated by the hyperbola $x^2=2z^2+\frac{1}{2}$. Hence the volume of the hyperboloid of one nappe generated $=\int \pi x^2 dz$, the upper limit being $\frac{1}{6}\sqrt{3}a$ and the lower limit $-\frac{1}{6}\sqrt{3}a$. This integral is $\frac{5}{27}\pi\sqrt{3}a^3$.



The lines AB, AE, and AD generate a cone, radius $=\frac{1}{3}\sqrt{6}a$, altitude $=\frac{1}{3}\sqrt{3}a$, volume $=\frac{2}{27}\pi\sqrt{3}a^3$.

The lines GF, GH, and GC generate another cone of the same size.

The sum of the volumes of the three solids $=\frac{1}{3}\pi\sqrt{3}a^3=1.8138a^3$.

[NOTE.—This solution by the Proposer is fuller than that given in the November number, and is published because several of our contributors failed to comprehend the abbreviated solution previously published. Prof. Whitaker asserts that the solution by Dr. Zerr in the September-October number is incorrect, while the latter says he does not as yet see Prof. Whitaker's hyperboloid. The above seems to be correct, but we shall be glad to have the criticisms of other contributors.—EDITOR.]

43. Proposed by Professor C. E. WHITE, A. M., Trafalgar, Indiana.

Prove that $\int_2^1 \frac{x^{a-1}+x^{-a}}{1+x}\frac{dx}{x}=\log(\tan\frac{a\pi}{2})$, when $a>0$ and <1.

[*Williamson's Integral Calculus, p. 154.*]

Comment by C. W. M. BLACK, A. M., Professor of Mathematics in Wesleyan Academy, Wilbraham, Massachusetts.

There seems to be an error in No. 43, as I find the following in my copy of *Williamson :*

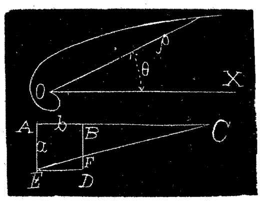
$$\int_0^1 \frac{x^{a-1}+x^{-a}}{1+x}\frac{dx}{\log x},$$

which gives the required result.

[In *Williamson's Integral Calculus*, edition of 1891, the problem is given as published, but the mistake has doubtless been corrected in the later edition.—EDITOR.]

44. Proposed by DE VOLSON WOOD, C. E., Professor of Mechanical and Electrical Engineering in Stevens Institute of Technology, Hoboken. New Jersey.

Find the equation of a curve in which $\rho=f(\theta)$, in which ρ is equal to BC, an intercept of any secant drawn from the corner E of the rectangle $AEDB$, and prolonged to cut AB prolonged in C. Let equal increments of θ be proportional to the equal incrementsof DB as divided by the secant EF, θ being zero when EC coincides with ED, and $\theta=2\pi$ when EF passes through B. Determine the asymptotes.



I. Solution by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pennsylvania; G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Applied Science, Texarkana College, Texarkana, Arkansas-Texas; and ALFRED HUME, C. E., D. Sc., Professor of Mathematics, University of Mississippi, University P. O., Mississippi.

Referring to the diagram given by the Proposer of this problem, July-Au-